JOURNAL
OF THE
WASHINGTON ACADEMY OF SCIENCES

VOL. 12 NOVEMBER 19, 1922 No. 19

MATHEMATICS.—*Values of sine θ and cosine θ to 33 places of decimals for various values of θ expressed in sexagesimal seconds.*[1] C. E. VAN ORSTRAND and MARVIN A. SHOULTES, Geological Survey.

One hundred preliminary values of both the sine and cosine were computed several years ago in collaboration with the late Dr. George F. Becker with the object in view of constructing complete tables of the natural values of the six trigonometric functions. In the meantime Professor J. Peters[2] published 870 values of the sine, and cosine, to 21 places of decimals, the interval of the argument for the first 10′ being 1″, and for the remainder of the semiquadrant, 10′. During the interval 1911–18, the elaborate tables to 15 places of decimals by Professor H. Andoyer[3] of the University of Paris were published. An unnumbered volume contains values of the logarithmic sines, cosines, tangents, and cotangents; and in volumes 1 to 3, respectively, are tabulated natural sines and cosines; tangents and cotangents; secants and cosecants. The interval of the argument in all of the tables is 10″. Volume 1 contains also natural values of each of the six trigonometric functions to 20 decimals for arguments expressed in hundredths of a quadrant. Andoyer's tables are the first to replace the great work of George Joachim Rheticus consisting of the Opus Palatinum, published in 1596 and containing 10 place values at intervals of 10″ for all of the trigonometric functions; and the Thesaurus Mathematicus which was published in 1613 and contains 15 place values of the natural sine at intervals of 10″ throughout the entire quadrant, and in addition, values of the same function to the same

[1] Published with the permission of the Director of the U. S. Geological Survey. Received June 30, 1922.

[2] PETERS, J. *Einundzwanzigstellige Werte der Funktionen Sinus und Cosinus zur genauen Berechnung von zwanzigstelligen Werten sämtlicher trigonometrischen Funktionen eines beliebigen Arguments sowie ihrer Logarithmen.* Abh. Königl. Preuss. Akad. Wiss. 1–54. 1911.

[3] ANDOYER, H. *Nouvelles tables trigonométriques fondamentales (logarithmes)*; same (*valeurs naturelles*).

number of decimals at intervals of 1″ for the first and last degree of the quadrant. Detailed information in regard to these and other tables is given by Andoyer, Glaisher,[4] and Horsburgh.[5]

The fundamental values required for the evaluation of the functions tabulated in the accompanying tables were obtained by the substitution of

$$x = 0.04848\ \ 13681\ \ 10953\ \ 59935\ \ 89914\ \ 10235\ \ 79479\ \ 760$$

in the series, thus obtaining values of sin θ and cos θ for 10,000″. The values for 1″, 10″, 100″, and 1000″ were then easily obtained from the preceding computations of $x^n/n!$ by making the appropriate displacements in the decimal point. With these values as a basis, new values were computed at equal intervals of the argument by means of repeated applications of the formulae for $sin\ (x + y)$ and $cos\ (x + y)$. Both functions were computed at the same time with a "Millionaire" computing machine. Each computation was verified independently. We have for example,

$$Sin\ (x \pm y) = sin\ x\ cos\ y \pm cos\ x\ sin\ y.$$

By addition of the last to the preceding term of this equation we obtain $sin\ (x + y)$, a new quantity; similarly by subtraction, we obtain $sin\ (x - y)$, a quantity previously computed. Apart from checks provided by each subsequent interpolation at equal intervals, comparisons were of course made at 15°, 30°, and 45° by means of the known values[6] of $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{6}$. All of the computations were carried to 35 places of decimals.

Andoyer's table 1, volume 1, contains expansions in series for each of the six trigonometric functions, the variable angle being written, $x\ \pi/2$. The coefficients in these series are tabulated to 24 places of decimals. In table 3, volume 1, each function is tabulated to 17 places of decimals at intervals of 9′. Tables I and II may be useful in connection with these tables and the tables of Peters previously mentioned. A careful comparison of our values with the corresponding values given by Peters and Andoyer revealed no errors in any of the computations. The plus signs following Andoyer's tabulations were also found to be given correctly.

[4] See article, *Table, mathematical,* in *Encyclopedia Britannica.*

[5] HORSBURGH, E. M. *Modern instruments and methods of calculation, a handbook of the Napier tercentenary exhibition.*

[6] BOORMAN, J. M. *Square-root notes.* Math. Mag. 1: 207–8. 1882–4. MARTIN, A. *Extraction of the square roots by series.* Math. Mag. 1: 164–5. 1882–4.

TABLE I.—VALUES OF SINE θ

θ''	Sine θ						
1	0.00000	48481	36811	07636	78200	79090	941
2	.00000	96962	73622	03878	29452	38417	064
3	.00001	45444	10432	77329	26805	60891	947
4	.00001	93925	47243	16594	43311	34785	949
5	0.00002	42406	84053	10278	52020	56404	615
6	.00002	90888	20862	46986	25984	32767	058
7	.00003	39369	57671	15322	38253	84284	359
8	.00003	87850	94479	03891	61880	47437	958
9	.00004	36332	31286	01298	69915	77458	047
10	0.00004	84813	68091	96148	35411	51001	965
11	.00005	33295	04896	77045	31419	68832	588
12	.00005	81776	41700	32594	30992	58496	724
13	.00006	30257	78502	51400	07182	77003	508
14	.00006	78739	15303	22067	33043	13502	792
15	0.00007	27220	52102	33200	81626	91963	541
16	.00007	75701	88899	73405	25987	73852	222
17	.00008	24183	25695	31285	39179	60811	203
18	.00008	72664	62488	95445	94256	97337	142
19	.00009	21145	99280	54491	64274	73459	381
20	0.00009	69627	36069	97027	22288	27418	340
21	.00010	18108	72857	11657	41353	48343	911
22	.00010	66590	09641	86986	94526	78933	846
23	.00011	15071	46424	11620	54865	18132	160
24	.00011	63552	83203	74162	95426	23807	513
25	0.00012	12034	19980	63218	89268	15431	611
26	.00012	60515	56754	67393	09449	76757	598
27	.00013	08996	93525	75290	29030	58498	445
28	.00013	57478	30293	75515	21070	81005	348
29	.00014	05959	67058	56672	58631	36946	118
30	0.00014	54441	03820	07367	14773	93983	577
31	.00015	02922	40578	16203	62560	97453	948
32	.00015	51403	77332	71786	75055	73045	251
33	.00015	99885	14083	62721	25322	29475	695
34	.00016	48366	50830	77611	86425	61172	069
35	0.00016	96847	87574	05063	31431	50948	141
36	.00017	45329	24313	33680	33406	72683	045
37	.00017	93810	61048	52067	65418	93999	675
38	.00018	42291	97779	48830	00536	78943	085
39	.00018	90773	34506	12572	11829	90658	871
40	0.00019	39254	71228	31898	72368	94071	575
41	.00019	87736	07945	95414	55225	58563	071
42	.00020	36217	44658	91724	33472	60650	959
43	.00020	84698	81367	09432	80183	86666	963
44	.00021	33180	18070	37144	68434	35435	318
45	0.00021	81661	54768	63464	71300	20951	167
46	.00022	30142	91461	76997	61858	75058	953
47	.00022	78624	28149	66348	13188	50130	812
48	.00023	27105	64832	20120	98369	21744	966
49	.00023	75587	01509	26920	90481	91364	117
50	0.00024	24068	38180	75352	62608	89013	840

TABLE 1.—VALUES OF SINE θ (*Continued*)

$\theta°$	Sine θ						
50	0.00024	24068	38180	75352	62608	89013	840
51	.00024	72549	74846	54020	87833	75960	974
52	.00025	21031	11506	51530	39241	47392	018
53	.00025	69512	48160	56485	89918	35091	522
54	.00026	17993	84808	57492	12952	10120	483
55	0.00026	66475	21450	43153	81431	85494	734
56	.00027	14956	58086	02075	68448	18863	341
57	.00027	63437	94715	22862	47093	15186	994
58	.00028	11919	31337	94118	90460	29416	399
59	.00028	60400	67954	04449	71644	69170	673
60	0.00029	08882	04563	42459	63742	97415	740
61	.00029	57363	41165	96753	39853	35142	716
62	.00030	05844	77761	55935	73075	64046	310
63	.00030	54326	14350	08611	36511	29203	213
64	.00031	02807	50931	43385	03263	41750	491
65	0.00031	51288	87505	48861	46436	81563	982
66	.00031	99770	24072	13645	39137	99936	684
67	.00032	48251	60631	26341	54475	22257	150
68	.00032	96732	97182	75554	65558	50687	882
69	.00033	45214	33726	49889	45499	66843	724
70	0.00033	93695	70262	37950	67412	34470	254
71	.00034	42177	06790	28343	04412	02122	178
72	.00034	90658	43310	09671	29616	05841	721
73	.00035	39139	79821	70540	16143	71837	023
74	.00035	87621	16324	99554	37116	19160	531
75	0.00036	36102	52819	85318	65656	62387	390
76	.00036	84583	89306	16437	74890	14293	839
77	.00037	33065	25783	81516	37943	88535	603
78	.00037	81546	62252	69159	27947	02326	285
79	.00038	30027	98712	67971	18030	79115	761
80	0.00038	78509	35163	66556	81328	51268	570
81	.00039	26990	71605	53520	90975	62742	310
82	.00039	75472	08038	17468	20109	71766	029
83	.00040	23953	44461	47003	41870	53518	620
84	.00040	72434	80875	30731	29400	02807	212
85	0.00041	20916	17279	57256	55842	36745	565
86	.00041	69397	53674	15183	94343	97432	460
87	.00042	17878	90058	93118	18053	54630	094
88	.00042	66360	26433	79664	00122	08442	474
89	.00043	14841	62798	63426	13702	91993	809
90	0.00043	63322	99153	33009	31951	74106	900
91	.00044	11804	35497	77018	28026	61981	538
92	.00044	60285	71831	84057	75088	03872	894
93	.00045	08767	08155	42732	46298	91769	913
94	.00045	57248	44468	41647	14824	64073	705
95	0.00046	05729	80770	69406	53833	08275	940
96	.00046	54211	17062	14615	36494	63637	242
97	.00047	02692	53342	65878	35982	23865	578
98	.00047	51173	89612	11800	25471	39794	654
99	.00047	99655	25870	40985	78140	22062	307
100	0.00048	48136	62117	42039	67169	43788	898

TABLE 1.—VALUES OF SINE θ (*Continued*)

θ''	Sine θ							θ °	′	″
100	0.00048	48136	32117	42039	67169	43788	898	0	01	40
200	.00096	96272	10282	15256	89823	20807	032	0	03	20
300	.00145	44405	30541	53507	62744	90817	111	0	05	00
400	.00193	92535	08942	96004	59253	71238	703	0	06	40
500	0.00242	40660	31533	89995	70314	77452	941	0	08	20
600	.00290	88779	84361	93442	43460	76882	902	0	10	00
700	.00339	36892	53474	77698	21461	61448	455	0	11	40
800	.00387	84997	24920	30186	80679	43000	289	0	13	20
900	.00436	33092	84746	57080	69045	76345	248	0	15	00
1000	0.00484	81178	19001	85979	43598	14483	948	0	16	40
2000	.00969	61216	85978	39588	10784	88351	447	0	33	20
3000	.01454	38976	51582	65677	03848	45733	109	0	50	00
4000	.01939	13317	71824	37245	47571	94786	397	1	06	40
5000	0.02423	83101	10748	13776	78016	39085	971	1	23	20
6000	.02908	47187	43111	40688	85777	50013	596	1	40	00
7000	.03393	04437	57062	23604	67590	79056	606	1	56	40
8000	.03877	53712	56816	71148	99343	73318	846	2	13	20
9000	.04361	93873	65335	99978	17530	77209	944	2	30	00
10000	0.04846	23782	27002	95750	84955	84328	790	2	46	40
11000	.05330	42300	10298	23748	20045	37349	674	3	03	20
12000	.05814	48289	10475	82853	87480	16847	071	3	20	00
13000	.06298	40611	52237	96604	80984	34293	964	3	36	40
14000	.06782	18129	92409	35025	77020	37025	599	3	53	20
15000	0.07265	79707	22610	60962	00827	96120	115	4	10	00
16000	.07749	24206	71930	94626	23706	99519	970	4	26	40
17000	.08232	50492	09599	90078	02676	95188	826	4	43	20
18000	.08715	57427	47658	17355	80642	70837	473	5	00	00
19000	.09198	43877	43627	43983	86954	31224	866	5	16	40
20000	0.09681	08707	03179	09579	14761	20018	367	5	33	20
21000	.10163	50781	82801	87285	02823	74772	066	5	50	00
22000	.10645	68967	92468	25762	15451	10176	599	6	06	40
23000	.11127	62131	98299	65468	94977	71117	421	6	23	20
24000	.11609	29141	25230	22967	56665	23380	711	6	40	00
25000	0.12090	68863	59669	36994	06114	61572	998	6	56	40
26000	.12571	80167	52162	70034	84187	81870	530	7	13	20
27000	.13052	61922	20051	59154	84062	27895	489	7	30	00
28000	.13533	12997	50131	09826	39365	73063	912	7	46	40
29000	.14013	32264	01306	26511	51356	19346	678	8	03	20
30000	0.14493	18593	07246	73754	06813	03177	897	8	20	00
31000	.14972	70856	79039	61542	36681	02368	752	8	36	40
32000	.15451	87928	07840	48706	78550	96980	932	8	53	20
33000	.15930	68680	67522	58121	33757	81353	360	9	10	00
34000	.16409	11989	17323	97482	52220	48582	552	9	26	40
35000	0.16887	16729	04492	79443	35126	33030	004	9	43	20

TABLE 1.—VALUES OF SINE θ (*Continued*)

θ''	Sine θ							θ °	′	″
35000	0.16887	16729	04492	79443	35126	33030	004	9	43	20
36000	.17364	81776	66930	34885	17166	26769	314	10	00	00
37000	.17842	06009	35832	13114	76243	73816	845	10	16	40
38000	.18318	88305	38326	62779	19399	48689	692	10	33	20
39000	.18795	27544	00111	87296	09103	66159	291	10	50	00
40000	0.19271	22605	48089	68602	24054	35580	049	11	06	40
41000	.19746	72371	12997	53028	93175	20166	624	11	23	20
42000	.20221	75723	32037	93118	00611	09131	373	11	40	00
43000	.20696	31545	51505	39198	33167	44529	770	11	56	40
44000	.21170	38722	29410	74548	29811	16675	828	12	13	20
45000	0.21643	96139	38102	87975	95536	69617	940	12	30	00
46000	.22117	02683	66887	77654	59084	24833	467	12	46	40
47000	.22589	57243	24644	80057	85664	86355	610	13	03	20
48000	.23061	58707	42440	17845	01983	49292	939	13	20	00
49000	.23533	05966	76137	60553	51441	77436	051	13	36	40
50000	0.24003	97913	09005	91962	72430	91690	162	13	53	20
51000	.24474	33439	54323	77999	82076	32813	311	14	10	00
52000	.24944	11440	57981	29065	51653	08864	737	14	26	40
53000	.25413	30812	01078	50664	78138	55503	119	14	43	20
54000	.25881	90451	02520	76234	88988	37624	048	15	00	00
55000	0.26349	89256	21610	76070	64197	81812	046	15	16	40
56000	.26817	26127	60637	36254	21023	94984	500	15	33	20
57000	.27283	99066	67461	01504	82378	06002	119	15	50	00
58000	.27750	09676	38095	75871	39833	40833	418	16	06	40
59000	.28215	54161	19287	75199	26412	51393	960	16	23	20
60000	0.28680	32327	11090	25310	32801	73167	158	16	40	00
61000	.29144	43081	69434	99844	33369	63357	123	16	56	40
62000	.29607	85334	08699	91717	35320	12347	692	17	13	20
63000	.30070	57995	04273	12162	25471	35931	073	17	30	00
64000	.30532	59976	95113	11324	64497	20020	700	17	46	40
65000	0.30993	90193	86305	14396	67978	45929	734	18	03	20
66000	.31454	47561	51613	67280	17265	82039	423	18	20	00
67000	.31914	30997	36030	85779	60933	62815	731	18	36	40
68000	.32373	39420	58321	02334	99482	31479	140	18	53	20
69000	.32831	71752	13561	04313	91905	07868	057	19	10	00
70000	0.33289	26914	75676	57891	82749	34745	209	19	26	40
71000	.33746	03832	99974	11559	22352	97522	744	19	43	20
72000	.34202	01433	25668	73304	40996	14682	259	20	00	00
73000	.34657	18643	78407	55530	39759	36664	117	20	16	40
74000	.35111	54394	72788	81774	76892	14440	613	20	33	20
75000	0.35565	07618	14876	49311	58452	17291	041	20	50	00
76000	.36017	77248	04710	41724	85846	71617	039	21	06	40
77000	.36469	62220	38811	85553	70672	05417	247	21	23	20
78000	.36920	61473	12684	45119	98878	30071	688	21	40	00
79000	.37370	73946	23310	49660	11760	53564	587	21	56	40
80000	0.37819	98581	71642	46893	70567	45860	696	22	13	20

TABLE 1.—VALUES OF SINE θ (*Continued*)

θ''	Sine θ							θ °	′	″
80000	0.37819	98581	71642	46893	70567	45860	696	22	13	20
81000	.38268	34323	65089	77172	84599	84030	399	22	30	00
82000	.38715	80118	20000	62367	09516	75664	442	22	46	40
83000	.39162	34913	64139	03650	63151	45548	608	23	03	20
84000	.39607	97660	39156	82369	60433	91609	744	23	20	00
85000	0.40052	67311	03060	58179	36996	58697	610	23	36	40
86000	.40496	42820	32673	58653	12675	97568	048	23	53	20
87000	.40939	23145	26092	54575	41387	94119	105	24	10	00
88000	.41381	07245	05139	15146	02720	61105	513	24	26	40
89000	.41821	94081	17806	37332	13027	39853	214	24	43	20
90000	0.42261	82617	40699	43618	69784	89647	730	25	00	00
91000	.42700	71819	81471	42419	92477	42377	872	25	16	40
92000	.43138	60656	81253	45426	86252	32813	699	25	33	20
93000	.43575	48099	17079	36179	31028	12022	586	25	50	00
94000	.44011	33120	04304	84162	88601	12690	874	26	06	40
95000	0.44446	14694	99020	98745	23555	28775	780	26	23	20
96000	.44879	91802	00462	17278	50403	34733	143	26	40	00
97000	.45312	63421	53408	21708	49345	06908	387	26	56	40
98000	.45744	28536	50580	78044	36287	95557	841	27	13	20
99000	.46174	86132	35033	93056	29306	73135	623	27	30	00
100000	0.46604	35197	02538	82582	23487	64495	630	27	46	40
101000	.47032	74721	03962	45838	69080	27959	114	28	03	20
102000	.47460	03697	47640	40144	44030	52184	505	28	20	00
103000	.47886	21122	01743	50480	21260	80880	521	28	36	40
104000	.48311	25992	96638	48321	53464	63171	169	28	53	20
105000	0.48735	17311	27242	34196	33658	08586	089	29	10	00
106000	.49157	94080	55370	58433	38248	28214	527	29	26	40
107000	.49579	55307	12079	14582	20902	62875	705	29	43	20
108000	.50000	00000	00000	00000	00000	00000	000	30	00	00
109000	.50419	27170	95670	38115	69879	97569	601	30	16	40
110000	0.50837	35834	51855	56896	66444	69809	366	30	33	20
111000	.51254	25007	99865	18058	30874	18895	538	30	50	00
112000	.51669	93711	51862	91572	41254	63063	465	31	06	40
113000	.52084	40968	03169	70045	20754	27285	637	31	23	20
114000	.52497	65803	34560	17551	82577	11220	776	31	40	00
115000	0.52909	67246	14552	47529	36243	83473	826	31	56	40
116000	.53320	44328	01691	24346	56755	94519	073	32	13	20
117000	.53729	96083	46823	83184	07855	46267	707	32	30	00
118000	.54138	21549	95369	62875	12861	67817	392	32	46	40
119000	.54545	19767	89582	46372	81410	58369	706	33	03	20
120000	0.54950	89780	70806	03526	27803	74050	134	33	20	00
121000	.55355	30634	81722	30864	56553	30355	275	33	36	40
122000	.55758	41379	68592	83103	43050	11746	238	33	53	20
123000	.56160	21067	83492	91107	02043	40781	882	34	10	00
124000	.56560	68754	86538	61053	13764	48220	929	34	26	40
125000	0.56959	83499	48106	49567	67013	60694	550	34	43	20

TABLE 1.—VALUES OF SINE θ (*Continued*)

θ''	Sine θ							θ °	′	″
125000	0.56959	83499	48106	49567	67013	60694	550	34	43	20
126000	.57357	64363	51046	09610	80319	12826	158	35	00	00
127000	.57754	10411	92885	01914	76330	96100	449	35	16	40
128000	.58149	20712	88026	66790	10886	48582	988	35	33	20
129000	.58542	94337	69940	51134	96645	10259	519	35	50	00
130000	0.58935	30360	93344	85499	11787	32161	758	36	06	40
131000	.59326	27860	36382	06072	46974	18896	909	36	23	20
132000	.59715	85917	02786	16485	18521	60583	960	36	40	00
133000	.60104	03615	24042	84324	62520	00059	672	36	56	40
134000	.60490	80042	61541	67292	23380	84844	332	37	13	20
135000	0.60876	14290	08720	63941	60975	42898	164	37	30	00
136000	.61260	05451	93202	83957	23105	40752	806	37	46	40
137000	.61642	52625	78925	32951	64466	48319	867	38	03	20
138000	.62023	54912	68260	06777	39492	34865	251	38	20	00
139000	.62403	11417	04126	90368	54453	15852	926	38	36	40
140000	0.62781	21246	72098	56145	33886	93521	289	38	53	20
141000	.63157	83513	02497	57034	37819	90534	395	39	10	00
142000	.63532	97330	72485	09175	59238	43787	042	39	26	40
143000	.63906	61818	08141	59406	35866	55041	224	39	43	20
144000	.64278	76096	86539	32632	26434	09907	263	40	00	00
145000	0.64649	39292	37806	54213	29246	73007	645	40	16	40
146000	.65018	50533	47183	42513	59944	14209	006	40	33	20
147000	.65386	08952	57069	66782	55812	60900	376	40	50	00
148000	.65752	13685	69063	65554	35855	19951	813	41	06	40
149000	.66116	63872	45993	20773	18972	59028	767	41	23	20
150000	0.66479	58656	13937	82870	87022	42597	499	41	40	00
151000	.66840	97183	64242	42043	75159	18990	214	41	56	40
152000	.67200	78605	55522	40995	78662	60690	329	42	13	20
153000	.67559	02076	15660	24434	83393	53674	354	42	30	00
154000	.67915	66753	43793	20629	56024	70184	810	42	46	40
155000	0.68270	71799	12292	50354	70231	73649	024	43	03	20
156000	.68624	16378	68733	58572	96049	99617	538	43	20	00
157000	.68975	99661	37857	64222	41556	11776	983	43	36	40
158000	.69326	20820	23524	23499	08872	00800	829	43	53	20
159000	.69674	79032	10655	02045	10164	03957	504	44	10	00
160000	0.70021	73477	67168	51473	83772	77449	050	44	26	40
161000	.70367	03341	45905	85684	55809	28499	420	44	43	20
162000	.70710	67811	86547	52440	08443	62104	849	45	00	00

TABLE 2.—VALUES OF COSINE θ

$\theta°$	Cosine θ						
1	0.99999	99999	88247	78473	04740	76217	925
2	.99999	99999	52991	13892	21725	33999	183
3	.99999	99998	94230	06257	59240	60726	215
4	.99999	99998	11964	55569	31098	02036	402
5	0.99999	99997	06194	61827	56633	61822	030
6	.99999	99995	76920	25032	60708	02230	248
7	.99999	99994	24141	45184	73706	43663	006
8	.99999	99992	47858	22284	31538	64776	987
9	.99999	99990	48070	56331	75639	02483	521
10	0.99999	99988	24778	47327	52966	51948	487
11	.99999	99985	77981	95272	16004	66592	202
12	.99999	99983	07681	00166	22761	58089	300
13	.99999	99980	13875	62010	36769	96368	596
14	.99999	99976	96565	80805	27087	09612	932
15	0.99999	99973	55751	56551	68294	84259	022
16	.99999	99969	91432	89250	40499	64997	267
17	.99999	99966	03609	78902	29332	54771	578
18	.99999	99961	92282	25508	25949	14779	165
19	.99999	99957	57450	29069	27029	64470	329
20	0.99999	99952	99113	89586	34778	81548	232
21	.99999	99948	17273	07060	56926	01968	655
22	.99999	99943	11927	81493	06725	19939	751
23	.99999	99937	83078	12885	02954	87921	772
24	.99999	99932	30724	01237	69918	16626	794
25	0.99999	99926	54865	46552	37442	75018	423
26	.99999	99920	55502	48830	40880	90311	490
27	.99999	99914	32635	08073	21109	47971	735
28	.99999	99907	86263	24282	24529	91715	471
29	.99999	99901	16386	97459	03068	23509	247
30	0.99999	99894	23006	27605	14175	03569	483
31	.99999	99887	06121	14722	20825	50362	106
32	.99999	99879	65731	58811	91519	40602	164
33	.99999	99872	01837	59876	00281	09253	432
34	.99999	99864	14439	17916	26659	49528	000
35	0.99999	99856	03536	32934	55728	12885	854
36	.99999	99847	69129	04932	78085	09034	439
37	.99999	99839	11217	33912	89853	05928	211
38	.99999	99830	29801	19876	92679	29768	179
39	.99999	99821	24880	62826	93735	65001	425
40	0.99999	99811	96455	62765	05718	54320	624
41	.99999	99802	44526	19693	46848	98663	540
42	.99999	99792	69092	33614	40872	57212	513
43	.99999	99782	70154	04530	17059	47393	936
44	.99999	99772	47711	32443	10204	44877	711
45	0.99999	99762	01764	17355	60626	83576	705
46	.99999	99751	32312	59270	14170	55646	175
47	.99999	99740	39356	58189	22204	11483	199
48	.99999	99729	22896	14115	41620	59726	082
49	.99999	99717	82931	27051	34837	67253	750
50	0.99999	99706	19461	96999	69797	59185	136

TABLE 2.—VALUES OF COSINE θ (*Continued*)

$\theta°$	Cosine						
50	0.99999	99706	19461	96999	69797	59185	136
51	.99999	99694	32488	23963	19967	18878	549
52	.99999	99682	22010	07944	64337	87931	033
53	.99999	99669	88027	48946	87425	66177	709
54	.99999	99657	30540	46972	79271	11691	105
55	0.99999	99644	49549	02025	35439	40780	481
56	.99999	99631	45053	14107	57020	27991	125
57	.99999	99618	17052	83222	50628	06103	652
58	.99999	99604	65548	09373	28401	66133	283
59	.99999	99590	90538	92563	08004	57329	106
60	0.99999	99576	92025	32795	12624	87173	336
61	.99999	99562	70007	30072	70975	21380	550
62	.99999	99548	24484	84399	17292	83896	920
63	.99999	99533	55457	95777	91339	56899	420
64	.99999	99518	62926	64212	38401	80795	034
65	0.99999	99503	46890	89706	09290	54219	942
66	.99999	99488	07350	72262	60341	34038	695
67	.99999	99472	44306	11885	53414	35343	377
68	.99999	99456	57757	08578	55894	31452	757
69	.99999	99440	47703	62345	40690	53911	423
70	0.99999	99424	14145	73189	86236	92488	905
71	.99999	99407	57083	41115	76491	95178	788
72	.99999	99390	76516	66127	00938	68197	809
73	.99999	99373	72445	48227	54584	75984	941
74	.99999	99356	44869	87421	37962	41200	462
75	0.99999	99338	93789	83712	57128	44725	018
76	.99999	99321	19205	37105	23664	25658	666
77	.99999	99303	21116	47603	54675	81319	908
78	.99999	99284	99523	15211	72793	67244	709
79	.99999	99266	54425	39934	06172	97185	505
80	0.99999	99247	85823	21774	88493	43110	195
81	.99999	99228	93716	60738	58959	35201	123
82	.99999	99209	78105	56829	62299	61854	047
83	.99999	99190	38990	10052	48767	69677	091
84	.99999	99170	76370	20411	74141	63489	687
85	0.99999	99150	90245	87911	99724	06321	506
86	.99999	99130	80617	12557	92342	19411	372
87	.99999	99110	47483	94354	24347	82206	163
88	.99999	99089	90846	33305	73617	32359	705
89	.99999	99069	10704	29417	23551	65731	647
90	0.99999	99048	07057	82693	63076	36386	323
91	.99999	99026	79906	93139	86641	56591	604
92	.99999	99005	29251	60760	94221	96817	736
93	.99999	98983	55091	85561	91316	85736	167
94	.99999	98961	57427	67547	88950	10218	353
95	0.99999	98939	36259	06724	03670	15334	564
96	.99999	98916	91586	03095	57550	04352	664
97	.99999	98894	23408	56667	78187	38736	887
98	.99999	98871	31726	67445	98704	38146	598
99	.99999	98848	16540	35435	57747	80435	036
100	0.99999	98824	77849	60641	99489	01648	050

TABLE 2.—VALUES OF COSINE θ (*Continued*)

θ''	Cosine θ							θ °	′	″
100	0.99999	98824	77849	60641	99489	01648	050	0	01	40
200	.99999	95299	11426	04859	14618	69087	718	0	03	20
300	.99999	89423	00812	19524	30450	81847	258	0	05	00
400	.99999	81196	46146	16090	05668	14138	802	0	06	40
500	0.99999	70619	47621	30585	47944	02697	103	0	08	20
600	.99999	57692	05486	23611	59460	12436	124	0	10	00
700	.99999	42414	20044	80335	52572	10605	823	0	11	40
800	.99999	24785	91656	10483	35623	63183	580	0	13	20
900	.99999	04807	20734	48331	68908	70286	798	0	15	00
1000	0.99998	82478	07749	52697	90782	60445	306	0	16	40
2000	.99995	29915	07226	15330	04692	98023	894	0	33	20
3000	.99989	42319	27107	52254	64532	27481	627	0	50	00
4000	.99981	19704	48501	39893	94719	26234	634	1	06	40
5000	0.99970	62090	04913	20525	42015	08166	159	1	23	20
6000	.99957	69500	82200	57696	26607	63405	281	1	40	00
7000	.99942	41967	18514	93368	05324	11432	941	1	56	40
8000	.99924	79525	04230	06928	80243	59014	703	2	13	20
9000	.99904	82215	81857	76240	37162	19403	330	2	30	00
10000	0.99882	50086	45950	40919	49149	63375	233	2	46	40
11000	.99857	83189	42990	68081	30755	65278	093	3	03	20
12000	.99830	81582	71268	20804	78207	08783	278	3	20	00
13000	.99801	45329	80743	29609	80108	59131	055	3	36	40
14000	.99769	74499	72897	67266	31651	24437	112	3	53	20
15000	0.99735	69167	00572	27286	33071	52804	468	4	10	00
16000	.99699	29411	67792	06480	00016	81034	629	4	26	40
17000	.99660	55319	29577	91987	59491	03072	423	4	43	20
18000	.99619	46980	91745	53229	50104	02473	888	5	00	00
19000	.99576	04493	10691	39246	89358	35953	664	5	16	40
20000	0.99530	27957	93165	81936	13607	09688	502	5	33	20
21000	.99482	17482	96033	05710	38033	04378	624	5	50	00
22000	.99431	73181	26018	44152	24463	29729	806	6	06	40
23000	.99378	95171	39442	64251	83970	77594	519	6	23	20
24000	.99323	83577	41942	98854	78955	52193	704	6	40	00
25000	0.99266	38528	88181	87975	25671	45608	280	6	56	40
26000	.99206	60160	81542	29659	32897	60508	774	7	13	20
27000	.99144	48613	73810	41114	45575	26928	563	7	30	00
28000	.99080	04033	64845	30850	93672	86613	503	7	46	40
29000	.99013	26572	02235	82611	76227	01884	454	8	03	20
30000	0.98944	16385	80944	51897	38370	64936	939	8	20	00
31000	.98872	73637	42938	75922	25125	21270	174	8	36	40
32000	.98798	98494	76808	97870	19733	65138	328	8	53	20
33000	.98722	91131	17374	06345	96272	07825	437	9	10	00
34000	.98644	51725	45273	90950	36130	58227	655	9	26	40
35000	0.98563	80461	86549	14936	75625	91705	341	9	43	20

TABLE 2.—VALUES OF COSINE θ (*Continued*)

θ''	Cosine θ							θ °	′	″
35000	0.98563	80461	86549	14936	75625	91705	341	9	43	20
36000	.98480	77530	12208	05936	67430	24589	523	10	00	00
37000	.98395	43125	37780	65772	51599	70530	389	10	16	40
38000	.98307	77448	22860	00405	42693	48954	215	10	33	20
39000	.98217	80704	70630	71096	47717	68768	433	10	50	00
40000	0.98125	53106	27384	67889	35337	51430	954	11	06	40
41000	.98030	94869	82024	06552	79906	21844	498	11	23	20
42000	.97934	06217	65551	50151	04288	24636	922	11	40	00
43000	.97834	87377	50547	56440	43137	64919	949	11	56	40
44000	.97733	38582	50635	52320	43159	80711	030	12	13	20
45000	0.97629	60071	19933	36597	08864	89605	428	12	30	00
46000	.97523	52087	52493	12346	91344	82532	502	12	46	40
47000	.97415	14880	81727	50199	03601	37006	148	13	03	20
48000	.97304	48705	79823	83883	28851	72784	696	13	20	00
49000	.97191	53822	57145	39421	67968	63170	943	13	36	40
50000	0.97076	30496	61619	99370	48705	41634	248	13	53	20
51000	.96958	78998	78116	03549	92542	09959	624	14	10	00
52000	.96838	99605	27805	87728	04796	72433	550	14	26	40
53000	.96716	92597	67516	61755	20007	10311	466	14	43	20
54000	.96592	58262	89068	28674	97431	99728	897	15	00	00
55000	0.96465	96893	18599	46367	20778	00334	742	15	16	40
56000	.96337	08786	15880	33308	11859	45689	290	15	33	20
57000	.96205	94244	73613	20062	19774	02922	903	15	50	00
58000	.96072	53577	16720	48149	95257	00092	449	16	06	40
59000	.95936	87097	01620	17965	04093	15820	317	16	23	20
60000	0.95798	95123	15488	87443	73747	66956	755	16	40	00
61000	.95658	77979	75512	23219	03657	05217	438	16	56	40
62000	.95516	35996	28123	06021	01829	41469	934	17	13	20
63000	.95371	69507	48226	92114	38470	64600	258	17	30	00
64000	.95224	78853	38415	32593	31211	28441	795	17	46	40
65000	0.95075	64379	28166	52382	96088	72745	030	18	03	20
66000	.94924	26435	73033	90826	13672	60314	740	18	20	00
67000	.94770	65378	53822	05762	70539	00165	703	18	36	40
68000	.94614	81568	75750	43038	42633	44150	010	18	53	20
69000	.94456	75372	67604	73408	88844	60829	404	19	10	00
70000	0.94296	47161	80875	98833	20272	84944	114	19	26	40
71000	.94133	97312	88887	30181	03151	04064	764	19	43	20
72000	.93969	26207	85908	38405	41092	77324	732	20	00	00
73000	.93802	34233	86257	81262	65235	97115	007	20	16	40
74000	.93633	21783	23393	07689	38851	27219	288	20	33	20
75000	0.93461	89253	48988	41975	56026	15508	867	20	50	00
76000	.93288	37047	32000	49900	82050	33097	417	21	06	40
77000	.93112	65572	57721	89030	46047	93683	301	21	23	20
78000	.92934	75242	26822	45395	54160	27564	276	21	40	00
79000	.92754	66474	54378	58810	44112	07511	693	21	56	40
80000	0.92572	39692	68890	39109	49227	43644	059	22	13	20

TABLE 2.—VALUES OF COSINE θ (*Continued*)

θ''	Cosine θ							°	′	″
80000	0.92572	39692	68890	39109	49227	43644	059	22	13	20
81000	.92387	95325	11286	75612	81831	89396	788	22	30	00
82000	.92201	33805	33918	42159	82417	51005	049	22	46	40
83000	.92012	55571	99539	00077	11892	01720	896	23	03	20
84000	.91821	61068	80274	01475	89614	15314	637	23	20	00
85000	0.91628	50744	56577	95301	99669	10379	085	23	36	40
86000	.91433	25053	16179	38590	01894	09911	083	23	53	20
87000	.91235	84453	53014	15400	92458	55321	636	24	10	00
88000	.91036	29409	66146	65950	61269	69701	259	24	26	40
89000	.90834	60390	58679	28464	90047	54815	463	24	43	20
90000	0.90630	77870	36649	96324	25526	56754	317	25	00	00
91000	.90424	82328	07917	93089	46829	87244	356	25	16	40
92000	.90216	74247	81037	68027	24559	96346	896	25	33	20
93000	.90006	54118	64121	14782	41492	26925	298	25	50	00
94000	.89794	22434	63688	15871	10879	25644	859	26	06	40
95000	0.89579	79694	83505	15696	88208	37774	595	26	23	20
96000	.89363	26403	23412	24819	25741	86866	655	26	40	00
97000	.89144	63068	78138	58231	66235	63600	156	26	56	40
98000	.88923	90205	36106	10433	12823	63874	580	27	13	20
99000	.88701	08331	78221	70105	46098	83037	517	27	30	00
100000	0.88476	17971	76657	77234	86857	64728	642	27	46	40
101000	.88249	19653	93621	25544	23738	08546	285	28	03	20
102000	.88020	13911	80111	13129	39007	65608	480	28	20	00
103000	.87789	01283	74664	44219	72983	15935	547	28	36	40
104000	.87555	82313	02090	85010	67925	63258	589	28	53	20
105000	0.87320	57547	72195	76542	25687	54324	677	29	10	00
106000	.87083	27540	78492	07624	99832	00274	584	29	26	40
107000	.86843	92849	96900	50841	22332	32894	675	29	43	20
108000	.86602	54037	84438	64676	37231	70752	936	30	00	00
109000	.86359	11671	77898	64861	78734	18410	727	30	16	40
110000	0.86113	66323	92513	68036	89047	11923	137	30	33	20
111000	.85866	18571	20613	10865	21839	08591	569	30	50	00
112000	.85616	68995	30266	47765	20353	68472	192	31	06	40
113000	.85365	18182	63916	30442	94966	76849	576	31	23	20
114000	.85111	66724	36999	72440	53230	15594	884	31	40	00
115000	0.84856	15216	36559	01939	56147	29489	679	31	56	40
116000	.84598	64259	19841	06085	87513	91409	795	32	13	20
117000	.84339	14458	12885	70127	28568	05827	572	32	30	00
118000	.84077	66423	09103	14682	27867	79545	790	32	46	40
119000	.83814	20768	67840	34483	46190	26250	481	33	03	20
120000	0.83548	78114	12936	41965	38261	70019	584	33	20	00
121000	.83281	39083	31267	19092	07224	03480	631	33	36	40
122000	.83012	04304	71278	80845	33858	88619	260	33	53	20
123000	.82740	74411	41510	53820	40664	30805	183	34	10	00
124000	.82467	50041	09106	73401	00852	96826	926	34	26	40
125000	0.82192	31835	98318	03011	44152	92055	530	34	43	20

TABLE 2.—VALUES OF COSINE θ (*Continued*)

θ''	Cosine θ							°	′	″
125000	0.82192	31835	98318	03011	44152	92055	530	34	43	20
126000	.81915	20442	88991	78968	44883	85916	843	35	00	00
127000	.81636	16513	15051	84481	03093	23590	596	35	16	40
128000	.81355	20702	62967	56371	46508	49682	892	35	33	20
129000	.81072	33671	70212	28115	89634	80549	570	35	50	00
130000	0.80787	56085	23711	12827	86442	99734	483	36	06	40
131000	.80500	88612	58278	29833	04691	39261	663	36	23	20
132000	.80212	31927	55043	78508	32948	91933	925	36	40	00
133000	.79921	86708	39869	63083	05777	40147	899	36	56	40
134000	.79629	53637	81755	72124	98229	69134	519	37	13	20
135000	0.79335	33402	91235	16457	97769	61501	299	37	30	00
136000	.79039	26695	18759	29283	09861	70580	147	37	46	40
137000	.78741	34210	53072	32298	92755	88165	015	38	03	20
138000	.78441	56649	19575	71641	47347	24386	379	38	20	00
139000	.78139	94715	78682	27488	09366	91314	549	38	36	40
140000	0.77836	49119	24160	01194	03499	36116	485	38	53	20
141000	.77531	20572	81465	83854	22268	34155	507	39	10	00
142000	.77224	09794	06069	10206	86630	78636	736	39	26	40
143000	.76915	17504	81765	01819	30109	64085	003	39	43	20
144000	.76604	44431	18978	03520	23926	50555	417	40	00	00
145000	0.76291	91303	53055	17066	26907	30247	919	40	16	40
146000	.75977	58856	42549	36054	00873	36254	422	40	33	20
147000	.75661	47828	67492	86112	79740	94670	113	40	50	00
148000	.75343	58963	27660	74436	18579	08569	961	41	06	40
149000	.75023	93007	40824	52733	77363	79999	078	41	23	20
150000	0.74702	50712	40995	97708	13061	51183	367	41	40	00
151000	.74379	32833	76661	13184	62921	28215	260	41	56	40
152000	.74054	40131	09004	58045	01400	11765	967	42	13	20
153000	.73727	73368	10124	04138	42933	94982	317	42	30	00
154000	.73399	33312	61235	28366	42745	16636	468	42	46	40
155000	0.73069	20736	50867	43161	17992	18861	828	43	03	20
156000	.72737	36415	73048	69598	71764	17663	816	43	20	00
157000	.72403	81130	25482	57411	52651	38384	266	43	36	40
158000	.72068	55664	07714	56187	22826	08402	286	43	53	20
159000	.71731	60805	19289	42062	37697	70109	592	44	10	00
160000	0.71392	97345	57899	04242	60206	36824	553	44	26	40
161000	.71052	66081	17520	95702	42639	44831	623	44	43	20
162000	.70710	67811	86547	52440	08443	62104	849	45	00	00

BOTANY.—*Ferns new to the Cuban Flora.*[1] WILLIAM R. MAXON, National Museum.

In the course of continued botanical exploration of eastern Cuba, Brother Léon collected extensively in the high Sierra Maestra, Oriente Province, during July of the present year and ascended Pico Turquino, the culminating point of the range, which has an altitude of approximately 2,300 meters. His collections were sent to the New York Botanical Garden, from which institution a set of the ferns has been forwarded to the National Museum for identification by the writer. Since the region is almost untouched botanically this material proves of exceptional interest, containing several new species as well as numerous others that are rare in Cuba or have been known heretofore only from the Blue Mountains of Jamaica. These are described and listed in the present paper.

The discovery of a pronounced montane Jamaican element at similar elevations in Cuba is not altogether unexpected, and the definite records of extended ranges here given will lead to a certain modification of the current idea of Jamaica as a highly endemic center, at least so far as ferns are concerned. With increasing collections from the high Sierra Maestra an analysis of this relation and of the elements common to Cuba and Hispaniola should prove of the highest interest from several points of view.

In addition to records afforded by Brother Léon's collection of 1922 there are included a few based upon material collected by Brother Léon and his associates a year or two earlier in other parts of the Sierra Maestra, mainly at a lower altitude.

CYATHEACEAE

Culcita coniifolia (Hook.) Maxon.

Near top of Pico Turquino, among shrubs (11155).

This is the only Cuban material seen by the writer. Known from Hispaniola and Jamaica, and on the continent from Mexico to Brazil and Ecuador.

Cyathea araneosa Maxon, N. Amer. Fl. 16: 74. 1909.

Sierra Maestra (11089).

Described from the Gran Piedra, Oriente, Cuba, on specimens collected by the writer (no. 4035), and since gathered by other collectors. The present specimen is the most completely fertile one seen, the segments throughout the pinnae all bearing 4 to 6 pairs of sori, instead of 1 to 3 basal pairs. The specific name was unfortunately chosen, inasmuch as the delicate, whitish-araneose inner border of the indusium, though evident enough in the

[1] Published by permission of the Secretary of the Smithsonian Institution. Received October 20, 1922.

type specimens, disappears at a very early stage, upon the ripening of the sporangia. Only the merest trace is discernible in the other specimens.

Cyathea producta Maxon, sp. nov.

Caudex erect, 4 to 6 meters high; fronds apparently ample; stipe 75 cm. long, olivaceous from a brown densely scurfy base, aculeate throughout, the spines numerous, 1 to 2 mm. long, conical, straight or antrorsely curved; basal part of stipe persistently paleaceous, the scales densely imbricate, 2 to 3.5 cm. long, acicular-caudate from a lanceolate base, atropurpureous, highly lustrous, with narrow yellowish brown, scarious, subentire borders; blade at least 1 meter broad, subtripinnate; rachis and middle pinnae wanting; basal pinnae oblong, 43 cm. long, 18 cm. broad, abruptly short-acuminate, long-petiolate (3 cm.), the secondary rachis dull olivaceous, closely yellowish-strigose above, beneath laxly and sparsely yellowish-hirsutulous, muricate toward the base; pinnules about 17 pairs, slightly apart, sessile or nearly so, 9 to 11 cm. long, 14 to 18 mm. broad, linear-oblong, in the apical third abruptly acuminate and conspicuously attenuate-caudate (the attenuate tip 2.5 to 3.5 cm. long), pinnatifid to about 1 mm. from the costa, the costa yellowish-strigose above, beneath laxly glandular-puberulous and deciduously paleaceous, the scales membranous, yellowish brown, broadly ovate, hair-pointed, subbullate, caducous; segments 14 to 16 pairs, oblong, rounded-obtuse, broadly connected by their strongly dilatate bases, 7 to 10 mm. long, 3.5 mm. broad at their middle, 5 mm. broad at base, spreading, falcate, membrano-herbaceous, glabrous above except for 1 to 3 stiff spinous hairs near the apex, thinly hirsute beneath along and between the veins to the freely long-ciliate, broadly crenate, slightly recurved margin; costules minutely paleaceous beneath, the scales like those of the costae; veins 8 or 9 pairs, acutely once forked near the base; sori 5 to 7 pairs, borne against the costa; indusia globose, transparent, membranous, rupturing irregularly; receptacle large, dark, depressed; paraphyses numerous, short, griseous-hyaline.

Type in the U. S. National Herbarium, no. 1,049,972, collected along a small stream near Palma Mocha Peak, Sierra Maestra, Oriente, Cuba, at 1,300 meters altitude, July, 1922, by Brother León (no. 11181).

In minute characters, such as the structure and disposition of hairs and scales upon the under surface of the blade, *C. producta* is most nearly related to *C. dissoluta* Baker, a rare plant of the Blue Mountain region of Jamaica, at 1,500 to 1,800 meters altitude. That species differs notably in having the pinnules closer, shorter (5 to 8 cm. long), and merely acuminate, the segments close, not at all dilatate, and not ciliate, the veins very oblique, partly simple, and running to obliquely crenate marginal teeth, and the scales of the costae and costules beneath highly colored and lustrous. Among related species *C. producta* is readily distinguished by its abruptly caudate pinnules alone.

Cyathea pubescens Mett.

Pico Turquino; trunk 4 to 6 meters tall (11151).

New to Cuba. Known heretofore from the mountains of Porto Rico, Hispaniola and Jamaica (the original region). Variable and perhaps needing segregation.

GLEICHENIACEAE

Dicranopteris furcata (L.) Underw.

Pico Turquino (11106).

The specimens are referred tentatively to this species, which Underwood

reports only from the Lesser Antilles (Martinique, Guadeloupe, and St. Kitts). The Cuban plant is essentially like recent Haitian specimens (*Leonard* 4273, 4273a), which will be discussed shortly in reporting on Mr. Leonard's collections.

Dicranopteris jamaicensis Underw. Bull. Torrey Club **34:** 258. 1907.

Northwestern spur of Pico Turquino, altitude 1,900 meters (11112).

Known hitherto only from Jamaica, where it is abundant at 1,600 to 2,225 meters, forming extensive thickets on the half-open ridges, along with *D. palmata*.

Dicranopteris leonis Maxon, sp. nov.

Plant of medium size, sparingly branched; rhizome not seen; primary leaf-axis (incomplete) olivaceous, mottled with brown, about 3 mm. thick, dull, deciduously scurfy-paleaceous. Primary branches apparently 1 or 2 pairs, 35 to 40 cm. long, twice pseudodichotomous, the included buds all dormant, the scales firm, up to 7 mm. long, narrowly deltoid, flexuous at the long-attenuate apex, bright castaneous and lustrous with pale borders, or paler and concolorous, the friable borders obliquely and laxly long-ciliate; first internode of the branches 2 to 3.5 cm. long, naked, subtended by 1 or 2 small segments; second internodes 2 to 8 cm. long, diverging at an angle of 60 to 80°, completely pectinate except at the outer side near the base, the lower segments gradually shorter; pinnae diverging at an angle of 60°, linear-lanceolate, 25 to 32 cm. long, 3.5 to 5 cm. broad, abruptly narrowed at the base, attenuate at the apex, pectinate throughout, cut nearly to the rachis, the sinuses linear, acute; rachises of pinnae brownish or (in the outer part) greenish, persistently paleaceous beneath, the scales spreading, light castaneous, firm, freely long-ciliate; segments 90 or more on each side, close, linear, 1.8 to 2.8 cm. long, 4 mm. broad at the base, 3 to 3.5 mm. broad at the middle, acutish or narrowly obtuse, herbaceous, minutely pale-papillate beneath; margins entire, narrowly revolute; veins 30 to 35 pairs, close, once forked, green, elevated both above and beneath, these and the strongly elevated, stramineous costae minutely paleaceous beneath, the scales castaneous, reduced, substellate, with firm spreading cilia, minute ones extending sparingly almost to the margin, 4 or 5 larger ones invariably borne close against the leaf surface from the base of the receptacle of the sori in a close radiating indusium-like group, the scales otherwise all distinct, never forming a tomentum or obscuring the leaf surface; sori mostly 4-sporangiate, inframedial.

Type in the U. S. National Herbarium, no. 1,049,896, collected in the Turquino region of the high Sierra Maestra, Oriente, Cuba, July, 1922, by Brother Léon (no. 11092). The description is partly drawn from another specimen of the same number in the Underwood Herbarium, New York Botanical Garden.

In gross structural characters *D. leonis* is not very unlike *D. palmata* (Schaffn.) Underw., *D. mellifera* (Christ) Underw., and *D. brittonii* Maxon, in all of which the stellate scales of the under surface are very greatly reduced and truly capillary, even flaccid. In the presence and distribution of firm true scales beneath it is nearer *D. longipinnata* (Hook.) Maxon, of Surinam, but the scales are utterly different in color and structure.[2]

[2] Cfr. Contr. U. S. Nat. Herb. **24**: 47–49. 1922.

POLYPODIACEAE

Elaphoglossum inaequalifolium (Jenman) C. Chr.

Pico Turquino, on trees (11166).

Described from Jamaica and known heretofore only from that island, where it is an abundant high-mountain species.

Polypodium gramineum Swartz.

Palma Mocha Peak, Sierra Maestra, at 1,400 meters elevation (11160).

Apparently not heretofore reported from Cuba. Agreeing closely with the present specimens are small plants collected on or near Pico Turquino by S. H. Hamilton in 1902. Both collections differ uniformly from the typical Jamaican plant in having much smaller and more delicate fronds, the stipes being more slender and the blades only 1 to 2 mm. broad, of thinner substance, and with some of the veins simple to the marginal connecting-vein; but in minute structural characters, such as those afforded by the rhizome scales and the once-forked glandular hairs of the leaf margin, the agreement is too close to justify the segregation of the Cuban plants as a distinct species. The extremes in width of blade do not, however, overlap. In Jamaica *P. gramineum* is abundant in the Blue Mountain region at 750 to 1,700 meters altitude.

Polypodium jenmani Underw.

East of Palma Mocha, Sierra Maestra, at 1,300 meters altitude, on tree trunks along a small stream (11101). Loma del Gato and vicinity, Cobre Range, Sierra Maestra, December, 1920, *Clement* 370.

New to Cuba, being known otherwise only from Jamaica, where it is very rare in the eastern part of the island at 500 to 900 meters elevation (*Maxon* 961, 1535; *Underwood* 2606; *Maxon & Killip* 173). The relationship is with *P. flexuosum* Maxon, of Cuba, which it resembles in scale structure.

Polypodium calvum Maxon, sp. nov.

Plants epiphytic, the fronds numerous, subfasciculate, rigidly ascending, 10 to 20 cm. long. Rhizome oblique or short-creeping, 1 cm. long or more, 3 or 4 mm. thick, coarsely long-radicose beneath, conspicuously paleaceous above, the scales ascending, loosely imbricate, 2.5 to 3 mm. long, 0.5 to 0.8 mm. broad, narrowly oblong-lanceolate, long-attenuate, attached just above the rounded subcordate base, entire, conspicuously clathrate, the cells with strongly sclerotic, dark reddish brown lateral walls, the outer walls pale yellowish, hyaline, greatly depressed; stipes short (0.5 to 1.5 cm. long), brown, nonsetose, bearing a few short branched glandular hairs, narrowly greenish-alate ventrally, the wings brownish with age; blades pinnatisect, 10 to 18 cm. long, 1 to 1.5 cm. broad near the middle, evenly attenuate in both directions, the apex not produced, the rachis nearly concealed above, beneath very prominent, black, lustrous, nonsetose, bearing a few branched glandular hairs, these evident mainly in the sinuses; pinnae 35 to 50 pairs, contiguous but not joined, spreading, the middle ones oblong or slightly triangular-oblong, broadest at base (3 to 4 mm.), rounded-obtuse or rarely acutish, rigidly coriaceous, nearly plane, entire, opaque, pale beneath, the venation wholly concealed; lower pinnae gradually shorter, triangular, the lowermost ones broader than long, short-decurrent; midveins of pinnae decurved at base, subflexuous; veins of larger pinnae 4 or 5 pairs, all but the proximal basal one diverging at about 45°, simple, ending in minute depressed-punctiform hydathodes remote from the margin; sori 3 or 4 pairs,

large, borne half way to the margin; sporangia numerous, glabrous, sometimes concealing 2 or 3 minute reddish setiform hairs.

Type in the U. S. National Herbarium, no. 1,049,931, collected in the high Sierra Maestra, Oriente, Cuba, in July, 1922, by Brother Léon (no. 11131). Collected also in the vicinity of Loma del Gato, Cobre Range, Sierra Maestra, altitude 1,100 meters, by Léon, Clement, and Roca (no. 10504).

A member of the group of *P. moniliforme* Lag., differing from the typical Jamaican form of that species in its short rhizome, its thick, short, rigid stipes, its numerous, mostly oblong pinnae, and its dark, heavily sclerotic rhizome scales. It is more nearly related to continental forms that are still erroneously retained in *P. moniliforme*.

Polypodium senile Fée.

Pico Turquino, on trees (11127).

New to the West Indian flora. Specimens are at hand from Costa Rica, Panama, Columbia, and Venezuela.[3]

Polypodium sherringii Baker.

Pico Turquino (11115).

Known hitherto only from the original collection, which came from the Newton District, Port Royal Mountains, Jamaica, at 1,200 to 1,500 meters altitude. This has recently been discussed by the writer.[4] The present specimens, which agree absolutely with a photograph of the type, show that the relationship with *P. basiattenuatum* Jenman is much more remote than previously supposed, Jenman's redescription[5] being accurate in every respect. The rigid, spongiose, dark green leaf-substance, with few, rigid, dark brown setae, the distant, oblique, decurrent lobes, and the decurrent foliaceous wing arising from the basal lobes at once distinguish this diminutive plant.

Cheilanthes harrisii Maxon, Contr. U. S. Nat. Herb. **24**: 51. 1922.

High Sierra Maestra (11184).

Known previously only from the vicinity of Cinchona, Jamaica, at 1,500 meters elevation. Allied to *C. marginata* H. B. K.

Paesia viscosa St. Hil.

Sierra Maestra, at 1,300 meters elevation; in woods (11159).

New to Cuba. An andine species of continental America, known hitherto in the West Indies only from the higher peaks of the Blue Mountains of Jamaica.

Psilogramme cubensis Maxon, sp. nov.

Rhizome stout, decumbent, 6 cm. long, 1.5 to 2 cm. thick (including the imbricate stipe-bases of old fronds), with numerous coarse wiry roots; rhizome hairs concealed, about 1.5 mm. long, dark purplish brown, opaque, rigid, simple, turgid, septate. Fronds several, clustered, arching, 40 to 60 cm. long; stipes nearly straight above the curved base, 20 to 28 cm. long, dark purplish brown, subscabrous, at first thinly short-villous with flattish septate hairs; blades oblong-lanceolate, acuminate, 20 to 32 cm. long, 8 to 13 cm. broad, subtripinnate, the primary rachis similar to the stipe, more freely villous, lightly flexuous throughout; larger primary pinnae 8 or 9 on each side, alternate, slightly oblique, subfalcate, inequilateral, those of the basal third the largest, triangular, acutish, 4 to 7 cm. long, 3 to 4.5 cm. broad,

[3] Cfr. Contr. U. S. Nat. Herb. **13**: 43. 1909.

[4] Contr. U. S. Nat. Herb. **17**: 552. 1916.

[5] Bull. Bot. Dept. Jamaica II. **4**: 113. 1897.

short-stalked (2 to 3 mm.), fully pinnate at base, the flexuous secondary rachis greenish-alate outward, bearing numerous pale, spreading, glistening, septate hairs; pinnules of larger pinnae 7 or 8 pairs below the pinnately lobed apex, pinnately parted, or the large basal ones fully pinnate at the base, with 4 or 5 pairs of spreading segments; larger segments in general broadly cuneate or cuneate-rhombic, 5 to 8 mm. long, 3 to 5 mm. broad (the base as broad as the common wing), obliquely cleft into 2 or 3 lobes, these simple or shallowly bilobate at tip; ultimate lobes 1.7 to 2 mm. broad; segments bright green, delicately membrano-herbaceous, bearing numerous stiff oblique tawny hairs above (both on and between the veins), the hairs of the lower side whitish, spreading, often gland-tipped, borne mainly on the veins; venation evident, the branches ending in the minutely emarginate tips of the lobes; sporangia relatively few, falling far short of the tips of the lobes, not long-decurrent.

Type in the U. S. National Herbarium, no. 1,049,913, collected on Pico Turquino, Sierra Maestra, Oriente, Cuba, in July, 1922, by Brother Léon (no. 11111).

A close ally of *Psilogramme chiapensis* Maxon,[6] of Mexico, belonging to the group of *P. hirta* and *P. glandulosa*, of South America. That species is similar to *P. cubensis* in structure, but differs in its much smaller pinnules and narrower segments (the ultimate lobes mostly 1 mm. broad, or less), and in its more copious hairy covering, this largely of a distinctly glandular type.

Asplenium diplosceuum Hieron. Hedwigia **60:** 232. 1918.

Loma del Gato and vicinity, Cobre Range, Sierra Maestra, at 1,000 meters altitude; in woods (*Léon, Clement & Roca* 10179).

Founded on Wright's no. 849, collected in some part of eastern Cuba, and known heretofore only on material of that collection.

Plagiogyria semicordata (Presl) Christ.

Near Pico Turquino (11126).

Apparently new to Cuba. It occurs on the summit of Blue Mountain Peak, Jamaica, altitude 2,225 meters, and on the continent from Mexico to the Andes of South America.

Struthiopteris shaferi Broadh. Bull. Torrey Club **39:** 374. 1912.

Slopes of Pico Turquino, at 1,800 to 1,900 meters altitude (11148).

Originally described from much smaller specimens, collected at Camp La Gloria, south of Sierra Moa, Oriente, Cuba, by J. A. Shafer (no. 8106). The present specimens are about 1 meter high and apparently represent a full development of the species. The sterile blade tapers gradually in the lower third, the lowermost pinnae being less than 1 mm. long. In other respects the plant agrees with the original material, the form of the sterile pinnae being especially distinctive.

Dryopteris grisebachii (Baker) Kuntze.

Near Palma Mocha, Sierra Maestra, at 1,300 meters altitude, along banks of small stream (11137). Vicinity of Loma del Gato, Cobre Range, Sierra Maestra, altitude 1,050 meters, in forest (*Léon, Clement & Roca* 10169).

Known from Cuba otherwise, apparently, only on Wright's no. 1055, the type collection. It occurs sparingly in the Blue Mountain region of Jamaica at elevations of 750 to 1,500 meters.

[6] Bull. Torrey Club **42:** 81. 1915.

Dryopteris hemiptera Maxon, Contr. U. S. Nat. Herb. **24:** 59. 1922.

Ridge of the Sierra Maestra, at 1,300 meters elevation, in forest (11133). Cobre range of the Sierra Maestra (*Léon, Clement & Roca* 10311, 10409, 10486).

When described this species of the subgenus *Stigmatopteris* was known only from the original collection (*Wright* 1053), from some part of eastern Cuba. No. 10486 is viviparous in the axils of most of the pinnae, bearing young plants with leaves 1 to 2.5 cm. long.

Dennstedtia globulifera (Poir.) Hieron. Bot. Jahrb. Engler **34:** 455. 1904.

Loma del Gato and vicinity, Cobre Range, Sierra Maestra, at 1,000 to 1,100 meters elevation (*Léon, Clement & Roca* 10177).

New to Cuba; originally described from Hispaniola. The history and relationship of this species will be discussed shortly in another connection.

HYMENOPHYLLACEAE

Hymenophyllum lineare Swartz.

Pico Turquino; on tree trunk (11116).

The only undoubted Cuban specimen of this species seen by the writer, though it has previously been ascribed to the island. Described originally from Jamaica, where it grows commonly in large mats on tree trunks of the upper forested slopes of the Blue Mountains, at 1,600 to 2,225 meters altitude. On the continent *H. lineare* is reported from Mexico to Brazil and Peru, but the name is very loosely used for several species of this group, which needs critical revision. The Cuban and Jamaican plants are identical.

LYCOPODIACEAE

Lycopodium montanum Underw. & Lloyd, Bull. Torrey Club **33:** 107. 1906.

Turquino region; terrestrial (11099).

Known previously only from the summit of Blue Mountain Peak, Jamaica, altitude 2,225 meters.

Lycopodium serratum Thunb.

Pico Turquino, in woods; terrestrial (11161).

A widely distributed Old World species known in America, hitherto, only from Mexico, whence it was described as *L. sargassifolium* Liebm.

PROCEEDINGS OF THE ACADEMY AND AFFILIATED SOCIETIES

BOTANICAL SOCIETY

157TH MEETING

The 157th meeting was held at the Cosmos Club, February 7, 1922. President SAFFORD was in the chair and 86 members and guests were present. Señor CARLOS A. VALLEJO, of the Argentine Embassy, was elected a member.

Dr. D. N. SHOEMAKER spoke of the death of Dr. WALTER VAN FLEET, formerly a member of the Botanical Society, and gave a résumé of his life and work. Dr. Van Fleet was born at Piermont, N. Y., June 18, 1857, and died at Miami, Florida, January 26, 1922. He was educated in medicine, and kept up his medical practice in Pennsylvania until 1893. From that time on his interests were largely in horticulture, particularly in the development and improvement by hybridization of hardy roses and gladioli, as well as of cannas, strawberries, and many other cultivated plants.

A. S. HITCHCOCK: *Botanical and agricultural notes from the Orient* (illustrated).

Dr. Hitchcock left San Francisco May 3 for Manila, stopping 6 days in Honolulu and arriving at his destination June 2. One month was spent on the island of Luzon, mostly at Manila, but a trip was also made to Los Baños, the seat of the Agricultural College, and Baguio in the mountains of the northern part of the island. The month of July was spent in Japan at Yokohama, Tokyo, Nikko, Kyoto, Lake Hakone and Mt. Fuji. From a botanical standpoint Mt. Fuji was disappointing.

During August Dr. Hitchcock visited Nanking in China, making his headquarters at the University of Nanking, and Kuling, a mountain resort much frequented by the missionaries of central China. About the first of September he went to Canton staying at the Canton Christian College. He made trips from here to Yingtak and Shiuchow on the North River, to Lohfau mountain northeast of Canton, to Whampoa, a place visited by the Wilkes Expedition, and to Macao, a Portuguese possession and the first locality occupied by Europeans in southeast Asia. Collections were also made at Hong Kong, the gateway to this part of China.

One of the most impressive things in Chinese agriculture was the extent to which the valleys were cultivated in the most intensive manner, while the hills or lands just above and adjoining the valleys which grow an abundance of grass, had no stock grazing upon them, partly on account of fear of bandits. This grass, however, did serve the purpose of fuel for cooking.

An excursion of about 5 weeks was made to Indo-China and the island of Hainan. French Indo-China was entered at Haiphong in Tonkin. Dr. Hitchcock went by rail to Hanoi and Vinh and by autobus to Hue, the capital of Annam. Here he made collections of grasses chiefly for the purpose of interpreting the work of Loureiro, a Portuguese botanist who lived here and in 1790 published a flora of Indo-China. After going to Tourane on the coast he returned to Haiphong and sailed for Hainan. This large island lies in the tropics off the south coast of China and is seldom visited and little known. About ten days were spent here mostly in making a trip to Kachek and the foothills of the central mountains. After returning to Canton he went to

Manila and sailed on an army transport November 15 for home, arriving in Washington the day before Christmas. Large collections of grasses were made in the countries visited as the season was favorable for this purpose.

Mr. G. N. COLLINS spoke on the Toronto Meeting of the American Association for the Advancement of Science, specially dealing with the papers on Genetics. These were about evenly divided between the botanists and zoologists. The leading number of papers concerned *Drosophila*, while the papers on maize ran second in number.

Dr. G. R. LYMAN spoke of the meeting of the Pathologists at the Toronto Meeting, and of the plan to coordinate the various biological scientists into one association similar to that of the Engineers Society and the Medical Society.

ROY G. PIERCE, *Recording Secretary.*

SCIENTIFIC NOTES AND NEWS

The educational courses at the Bureau of Standards this winter include: Harmonic Series Applied to Physical Problems, D. R. HARPER, 3D; Advanced Organic Chemistry, L. L. STEELE; Physical Metallurgy, H. S. RAWDON.

JOHN L. BAER, acting curator of American Archeology, National Museum, during the past few months, is in Pennsylvania, continuing his study of aboriginal quarry sites in the Shenandoah Valley.

Dr. ALEXANDER GRAHAM BELL, a resident member of the ACADEMY, died at his summer home in Nova Scotia on August 2, 1922, in his seventy-sixth year. Dr. Bell was born at Edinburgh, Scotland, on March 3, 1847. His invention of the telephone stands as his most notable scientific achievement, but he carried on research in various subjects, as for example, animal breeding, relief of deafness, applications of electricity, etc. He was a member of the ACADEMY, the Anthropological, Historical, and Philosophical Societies of Washington, and the Institute of Electrical Engineers.

HENRY B. COLLINS, JR., of Louisiana, a member of the 1922 National Geographic Society expedition to Pueblo Bonito, will spend the next few months in Washington studying the collections of the expedition.

MAYO D. HERSEY, formerly of the Bureau of Standards, and until recently associate professor of properties of matter in the department of physics, Massachusetts Institute of Technology, has resigned to take a position as physicist in charge of the physical laboratory of the U. S. Bureau of Mines at Pittsburgh, Pa.

NEIL M. JUDD, curator of American Archeology at the National Museum, returned to Washington September 28, following completion of the second season's explorations at Pueblo Bonito, under the auspices of the National Geographic Society. About forty secular rooms and five kivas were excavated during the summer and, in addition, considerable attention was devoted to study of the possible geophysical conditions which prevailed in Chaco Canyon at the time the great ruin was occupied.

Dr. SAMUEL W. STRATTON, director of the U. S. Bureau of Standards since its foundation in 1901, has resigned to become president of the Massachusetts Institute of Technology.

Dr. F. E. WRIGHT of the Geophysical Laboratory, Carnegie Institution of Washington, has returned after a year's absence in South Africa, where he has been engaged in geological and petrological investigations in company with Messrs. DALY, PALACHE, and MOLENGRAAF.